

*A*  
**COURSE FILE**  
*ON*  
**“DISCRETE MATHEMATICS”**

**II B-Tech I Semester (Autonomous)**



**COMPUTER SCIENCE & ENGINEERING**

**CMR ENGINEERING COLLEGE**

**KANDLAKOYA (V), MEDCHAL (M), R.R.DIST.**

**(ACADEMIC YEAR      2022-23)**

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**Submitted By**

# 1. DEPARTMENT VISION & MISSION

## Vision:

To produce globally competent and industry-ready graduates in Computer Science & Engineering by imparting quality education with the know-how of cutting-edge technology and holistic personality.

## Mission:

1. To offer high-quality education in Computer Science & Engineering in order to build core competence for the graduates by laying a solid foundation in Applied Mathematics and program framework with a focus on concept building.

2. The department promotes excellence in teaching, research, and collaborative activities to prepare graduates for a professional career or higher studies.

3. Creating an intellectual environment for developing logical skills and problem-solving strategies, thus developing, an able and proficient computer engineer to compete in the current global scenario.

# 2. LIST OF PEOs, POs AND PSOs

## 2.1 Program Educational Objectives (PEO):

**PEO1:** Excel in professional career and higher education by acquiring knowledge of mathematical computing and engineering principles.

**PEO2:** To provide an intellectual environment for analyzing and designing computing systems for technical needs.

**PEO3:** Exhibit professionalism to adapt current trends using lifelong learning with legal and ethical responsibilities.

**PEO4:** To produce responsible graduates with effective communication skills and multidisciplinary practices to serve society and preserve the environment.

## 2.2. Program Outcomes (POs):

Engineering Graduates will be able to satisfy these NBA graduate attributes:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems
2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations

4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice
9. **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change

### 2.3 Program Specific Outcomes (PSOs):

**PSO1: Professional Skills and Foundations of Software development:** Ability to analyze, design and develop applications by adopting the dynamic nature of Software developments.

**PSO2: Applications of Computing and Research Ability:** Ability to use knowledge in cutting edge technologies in identifying research gaps and to render solutions with innovative ideas.

## 3. COURSE OUTCOMES

- CO1• Understand and **construct** precise mathematical proofs (Apply)
- CO2• Ability to use logic and set theory to **formulate** precise statements (Create)
- CO3• **Analyze** and solve counting problems on finite and discrete structures (Analyze)
- CO4• **Elaborate** and manipulate sequences and advanced computing techniques (Create)
- CO5• **Apply** graph theory in solving computing problems (Apply)

## **4. SYLLABUS COPY**

### **UNIT - I**

The Foundations: Logic and Proofs: Propositional Logic, Applications of Propositional Logic, Propositional Equivalence, Predicates and Quantifiers, Nested Quantifiers, Rules of Inference, Introduction to Proofs, Proof Methods and Strategy.

### **UNIT - II**

Basic Structures, Sets, Functions, Sequences, Sums, Matrices and Relations Sets, Functions, Sequences & Summations, Cardinality of Sets and Matrices Relations, Relations and Their Properties, n-ary Relations and Their Applications, Representing Relations, Closures of Relations, Equivalence Relations, Partial Orderings.

### **UNIT – III**

Algorithms, Induction and Recursion: Algorithms, The Growth of Functions, Complexity of Algorithms Induction and Recursion: Mathematical Induction, Strong Induction and Well-Ordering, Recursive Definitions and Structural Induction, Recursive Algorithms, Program Correctness

### **UNIT – IV**

Discrete Probability and Advanced Counting Techniques: An Introduction to Discrete Probability, Probability Theory, Bayes' Theorem, Expected Value and Variance Advanced Counting Techniques: Recurrence Relations, Solving Linear Recurrence Relations, Divide-and-Conquer Algorithms and Recurrence Relations, Generating Functions, Inclusion-Exclusion, Applications of Inclusion-Exclusion

### **UNIT - V**

Graphs: Graphs and Graph Models, Graph Terminology and Special Types of Graphs, Representing Graphs and Graph Isomorphism, Connectivity, Euler and Hamilton Paths, Shortest-Path Problems, Planar Graphs, Graph Coloring. Trees: Introduction to Trees, Applications of Trees, Tree Traversal, Spanning Trees, Minimum Spanning Trees

### **TEXT BOOK:**

1. Discrete Mathematics and its Applications with Combinatorics and Graph Theory- Kenneth H Rosen, 7th Edition, TMH.

## **REFERENCES BOOKS:**

1. Discrete Mathematical Structures with Applications to Computer Science-J.P. Tremblay and R. Manohar, TMH,
2. Discrete Mathematics for Computer Scientists & Mathematicians: Joe L. Mott, Abraham Kandel, Theodore P. Baker, 2nd ed, Pearson Education.
3. Discrete Mathematics- Richard Johnsonbaugh, 7Th Edn., Pearson Education.
4. Discrete Mathematics with Graph Theory- Edgar G. Goodaire, Michael M. Parmenter.
5. Discrete and Combinatorial Mathematics - an applied introduction: Ralph.P. Grimald, 5th edition, Pearson Education.

## 5. SESSION PLAN/LESSON PLAN

S.NO	Topic (JNTU syllabus)	Sub-Topic	NO. OF LECTURES REQUIRED	Suggested Books	Teaching Methods
UNIT – I					
1	The Foundations: Logic and Proofs	Propositional Logic	L1	T1	M1
2		Applications of Propositional Logic	L2-L3	T1	M1
3		Propositional Equivalence	L4-L5	T1	M2(PPT)
4		Predicates and Quantifiers	L6-L7	T1	M2(PPT)
5		Nested Quantifiers	L8, L9	T1	M2(E-resources)
6		Rules of Inference.	L10, L11	T1	M2(E-resources)
7		Introduction to Proofs	L12, L13	T1	M1
8		Proof Methods and Strategy	L14, L15	T1	M1
UNIT-II					
9	Basic Structures, Sets	Basic Structures	L16	T1	M1
10		Sets	L17	T1	M1
11		Functions	L18	T1	M2(E-resources)
12		Sums, Matrices and Relations Sets	L19	T1	M2(E-resources)
13		Sequences & Summations	L20	T1	M1
14		Cardinality of Sets and Matrices Relations	L21	T1	M1
15		Relations and Their Properties	L22	T1	M1
16		n-ary Relations and Their Applications	L23	T1	M1
17		Representing Relations, Closures of Relations	L24	T1	M2(E-resources)

18		Equivalence Relations, Partial Orderings	L25,L26	T1	M2(E-resources)
UNIT III					
19	Algorithms,  Induction and Recursion	Algorithms,	L27	T1	M1
20		The Growth of Functions	L28	T1	M2(E-resources)
21		Complexity of Algorithms	L29	T1	M2(E-resources)
22		Mathematical Induction	L30	T1	M2(E-resources)
23		Strong Induction and Well-Ordering	L31	T1	M1
24		Recursive Definitions	L32	T1	M1
25		Structural Induction	L33	T1	M1
26		Recursive Algorithms	L34	T1	M1
27		Program Correctness	L35	T1	M1
UNIT-IV					
28	Discrete Probability and Advanced Counting Techniques	An Introduction to Discrete Probability	L36	T1	M1
29		Probability Theory, Bayes' Theorem	L37	T1	M1
30		Expected Value and Variance	L38	T1	M2(PPT)
31		Recurrence Relations	L39	T1	M2(NPTEL)
32		Solving Linear Recurrence Relations	L40	T1	M2(NPTEL)
33		Divide-and-Conquer Algorithms and Recurrence Relations	L41	T1	M2(E-resources)
34		Generating Functions	L42	T1	M1
35		InclusionExclusion	L43	T1	M1
36		Applications of Inclusion-Exclusion	L44	T1	M1
UNIT-V					



<b>37</b>	<b>Graphs &amp; Trees</b>	Graphs and Graph Models	<b>L45</b>	<b>T1</b>	<b>M1</b>
<b>38</b>		Graph Terminology and Special Types of Graphs	<b>L46</b>	<b>T1</b>	<b>M2(PPT)</b>
<b>39</b>		Representing Graphs and Graph Isomorphism, Connectivity	<b>L47</b>	<b>T1</b>	<b>M2(NPTEL)</b>
<b>40</b>		Euler and Hamilton Paths, Shortest-Path Problems	<b>L48-L49</b>	<b>T1</b>	<b>M2(PPT)</b>
<b>41</b>		Planar Graphs, Graph Coloring	<b>L50</b>	<b>T1</b>	<b>M1</b>
<b>42</b>		Introduction to Trees, Applications of Trees	<b>L51-L52</b>	<b>T1</b>	<b>M2(NPTEL)</b>
<b>43</b>		Tree Traversal	<b>L53</b>	<b>T1</b>	<b>M1</b>
<b>44</b>		Spanning Trees,	<b>L54</b>	<b>T1</b>	<b>M1</b>
<b>45</b>		Minimum Spanning Trees	<b>L55</b>	<b>T1</b>	<b>M1</b>

#### **METHODS OF TEACHING:**

<b>M1 : Lecture Method</b>	<b>M4 : Presentation /PPT</b>	<b>M7 : Assignment</b>
<b>M2 : Demo Method</b>	<b>M5 : Lab/Practical</b>	<b>M8 : Industry Visit</b>
<b>M3 : Guest Lecture</b>	<b>M6 : Tutorial</b>	<b>M9 : Project Based</b>

#### **NOTE:**

1. Any Subject in a Semester is suppose to be completed in 55 to 65 periods.
2. Each Period is of 50 minutes.
3. Each unit duration & completion should be mentioned in the Remarks Coloumn.
4. List of Suggested books can be marked with Codes like T1, T2, R1, R2 etc.

## INDIVIDUAL TIME TABLE(PRATHIMA Y)

	I	II	III	IV		V	VI	VII
MON		DM-D				DM-B	DM-B	
TUE								
WED								
THU								
FRI			DM-B	DM-D				
SAT	DM-B	DM-D				DM-D		

### 6. Session Execution Log:

S no	Units	Scheduled started date	Completed date	Remarks
1	I	10-10-22	22-10-22	COMPLETED
2	II	25-10-22	19-11-22	COMPLETED
3	III	22-11-22	26-12-22	COMPLETED
4	IV	23-12-22	10-1-23	COMPLETED
5	V	12-1-23	25-1-23	COMPLETED

## **7. Lecture Notes – (hand written)**

## 8. ASSIGNMENT QUESTIONS ALONG SAMPLE ASSIGNMENT SCRIPTS



### CMR ENGINEERING COLLEGE

(Accredited by NBA, Approved by AICTE, Affiliated to JNTU, Hyderabad)

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### Department of Computer Science & Engineering

#### II.B.TECH, I SEM - I MID ASSIGNMENT QUESTIONS

**Subject: DISCRETE MATHEMATICS**

**BRANCH: CSE -B**

#### Answer the following questions

1. Let  $p$  and  $q$  be the propositions  
 $p$  : You drive over 65 miles per hour.  
 $q$  : You get a speeding ticket.

Write these propositions using  $p$  and  $q$  and logical connective (including negations).

- a) You do not drive over 65 miles per hour.
- b) You drive over 65 miles per hour, but you do not get a speeding ticket.
- c) You will get a speeding ticket if you drive over 65 miles per hour.
- d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
- e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
- f) You get a speeding ticket, but you do not drive over 65 miles per hour.
- g) Whenever you get a speeding ticket, you are driving over 65 miles per hour

2. Let  $p$  and  $q$  be the propositions  
“Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively.  
Express each of these compound propositions as an English sentence.

- a)  $\neg q$
- b)  $p \wedge q$
- c)  $\neg p \vee q$
- d)  $p \rightarrow \neg q$
- e)  $\neg q \rightarrow p$
- f)  $\neg p \rightarrow \neg q$
- g)  $p \leftrightarrow \neg q$
- h)  $\neg p \wedge (p \vee \neg q)$

3. Show that each of these conditional statements is a tautology.

- a)  $(p \wedge q) \rightarrow p$
- b)  $p \rightarrow (p \vee q)$
- c)  $\neg p \rightarrow (p \rightarrow q)$
- d)  $(p \wedge q) \rightarrow (p \rightarrow q)$
- e)  $\neg(p \rightarrow q) \rightarrow p$
- f)  $\neg(p \rightarrow q) \rightarrow \neg q$
- g)  $[\neg p \wedge (p \vee q)] \rightarrow q$
- h)  $[p \wedge (p \rightarrow q)] \rightarrow q$

## 9. MID EXAM QUESTION PAPER ALONG SAMPLE ANSWER SCRIPTS



**CMR ENGINEERING COLLEGE**  
**UGC AUTONOMOUS**

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Kandlakoya (V), Medchal (M), Medchal - Malkajgiri (D)-501401



**IL.B.TECH- I-SEM-I MID EXAMINATIONS**     **Date:** 12/11/21 **Time:** 10:00am to 11:30am

**Subject: Discrete Mathematics**

**Branch: Common to CSE/ IT/ AIML**

**Marks: 25 M**

### **PART-A**

**5x2=10**

**1 Q.**(a) Let  $p, q, r$  denote the following statements about a triangle ABC.

$p$  : Triangle ABC is isosceles;

$q$  : Triangle ABC is equilateral;

$r$  : Triangle ABC is equiangular.

Translate each of the following into an English sentence.

i).  $q \rightarrow p$      (ii).  $\neg p \rightarrow \neg q$      (iii).  $q \leftrightarrow r$      (iv).  $p \wedge \neg q$      (v).  $r \rightarrow p$

b) Write the converse, inverse, contrapositive, and negation of the following statement.

“If Sandra finishes her work, she will go to the basketball game.”

(CO1)

**2 Q.** Find DNF of the following:

(i)  $\sim(p \vee q) \rightarrow (p \wedge q)$

(ii)  $p \rightarrow [(p \rightarrow q) \wedge (\sim q \vee \sim p)]$

(CO1)

**3 Q.** List the different types of functions and explain with neat sketch.

(CO2)

**4 Q.** Discuss about the Properties of the Relations

(CO2)

**5 Q.** Define Pseudocode.

(CO3)

### **PART-B**

**3X5=15**

**6 Q.**(a) Show that the proposition:  $(P \vee \neg Q) \wedge (\neg P \vee Q) \vee Q$  is a tautology.

(b) Prove or Disprove the following statements are logically Equivalent without Truth Tables:

$(p \rightarrow q) \wedge (p \rightarrow r)$  and  $p \rightarrow (q \wedge r)$

(CO1)

(OR)

7 Q.(a)Construct the truth table for the following propositions :

i)( $p \rightarrow q$ )  $\wedge$  ( $\neg p \rightarrow q$ )   ii)  $p \rightarrow (\neg q \vee r)$

(b)Determine whether the conclusion follows logically from the premises. Explain by representing the statements symbolically and using rules of inference.Are the following arguments logically correct?

.Consider, Premises: If there was a ball game, then traveling was difficult.

If they arrived on time, then traveling was not difficult.

They arrived on time.

Conclusion: There was no ball game. (CO1)

a)Define Cardinality

b) Let f and g be functions from the positive real numbers to positive real numbers defined by

i.  $f(x) = \lfloor 2x \rfloor$

ii.  $g(x) = x^2$

Calculate  $f \circ g$  and  $g \circ f$ . (CO2)

(OR)

9 Q .What is Hasse diagram? Explain with an example and neat sketch. (CO2)

10 Q. a)Discuss about the Partial Ordering and POSET.

b)Discuss whether the following set  $S = \{1,2,3,4,5,6\}$  and the relation  $R = \{(i,j) : i-j=2\}$  is Partially Ordered or not?

(CO2)

(OR)

11 Q.What is Time complexity of an Algorithm,discuss about the properties of Algorithms. (CO3)

## 10. Mid-1 Scheme of evaluation

### DEPARTMENT OF CSE

COURSE: **B.Tech**

YEAR: **II**

SEM: **I**

A-Y: **2021-22**

NAME OF SUBJECT: **DISCRETE MATHEMATICS**

MID: **I**

Max marks: **30**

### DESCRIPTIVE TEST

Time : 10.00 AM TO 11-30 AM.

Part A questions are all mandatory. Part B attempt any 3 question from any unit

### SCHEME OF EVALUATION

S.No	THEORY	MARKS	TOTAL
Part-A	<p>Let p, q, r denote the following statements about a triangle ABC.</p> <p>p : Triangle ABC is isosceles;</p> <p>q : Triangle ABC is equilateral;</p> <p>r : Triangle ABC is equiangular.</p> <p>a Translate each of the following into an English sentence.</p> <p>i). <math>q \rightarrow p</math> (ii). <math>\neg p \rightarrow \neg q</math> (iii). <math>q \leftrightarrow r</math>            (iv). <math>p \wedge \neg q</math> (v). <math>r \rightarrow p</math></p> <p>Write the converse, inverse, contrapositive, and negation of the following statement.</p> <p>“If Sandra finishes her work, she will go to the basketball game.”.</p>	1	2
	<p>b Find DNF of the following:</p> <p>(i) <math>\sim(p \vee q) \rightarrow (p \wedge q)</math></p> <p>(ii) <math>p \rightarrow [(p \rightarrow q) \wedge (\sim q \vee \sim p)]</math></p>	1	2
	c List the different types of functions and explain with neat sketch.	2	2
	d Discuss about the Properties of the Relations	2	2
	e Define Pseudocode	2	2

Part-B	Q1.	(a) Show that the proposition: (P ∨ 7Q) ∧ (7P ∨ 7Q) ∨ Q is a tautology.	2.5	5	15Marks
		(b) Prove or Disprove the following statements are logically Equivalent without Truth Tables:  (p → q) ∧ (p → r) and p → (q ∧ r)	2.5		
	Q2.	(a) Construct the truth table for the following propositions :  (p → q) ∧ (7p → q)    ii)   p → (7q ∨ r)	2.5	5	
		(a) Determine whether the conclusion follows logically from the premises. Explain by representing the statements symbolically and using rules of inference. Are the following arguments logically correct?  Consider, Premises: If there was a ball game, then traveling was difficult.  If they arrived on time, then traveling was not difficult.  They arrived on time.  Conclusion: There was no ball game.	2.5		
		Cardinality  Let f and g be functions from the positive real numbers to positive real numbers defined by  i.     f(x) = ⌊ 2x ⌋  ii.     g(x) = x <sup>2</sup>  Calculate fOg and gOf.	5		
	Q4	What is Hasse diagram? Explain with an example and neat sketch.	5	5	
Q5.	a) Discuss about the Partial Ordering and POSET.	2.5	5		
	b) Discuss whether the following set S=	2.5			



		{1,2,3,4,5,,6} and the relation $R=\{ (i,j): i-j=2 \}$ is Partially Ordered or not?			
	<b>Q6</b>	What is Time complexity of an Algorithm , discuss about the properties of Algorithms	5	5	
		<b>DESCRIPTIVE TEST</b>			25 Marks
		<b>ASSIGNMENT</b>			5Marks
		<b>TOTAL</b>			30 Marks



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Kandlakoya (V), Medchal (M), Medchal - Malkajgiri (D)-501401



**II.B.TECH- I-SEM-II MID EXAMINATIONS,**

**Date:**

**Time: 10:00am to 11:30am**

**Subject: Discrete Mathematics**

**Branch: Common to CSE/ IT/ AIML**

**Marks: 25 M**

**Note: Question paper contains two parts, Part - A and Part - B.**

**Part-A is compulsory which carries 10 marks. Answer all questions in part-A.**

**Part-B consists of (2½) units. Answer any one full question from each unit. Each question carries 5 marks and may have a,b,c sub questions.**

### PART-A

**5x2=10**

**1 Q.** Explain about the Strong Induction

(CO3)

**2 Q.** Discuss the following terms: (i) mean (ii) variance

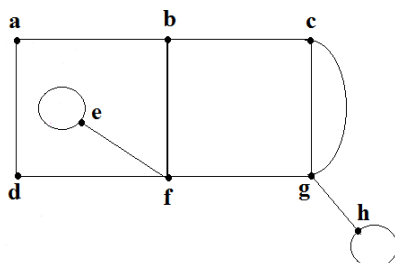
(CO4)

**3 Q.** State and prove the Bayes Theorem?

(CO4)

**4 Q. a)** Define Spanning tree .

**b)** Find the degree of each region in the following Planar graph



(CO5)

5 Q. Define Bipartite graph. Discuss with an example.

(CO5)

**PART-B**

**3X5=15**

6 . Prove by Mathematical induction that  $6^{n+2} + 7^{2n+1}$  is divisible by 43 for each positive integer n. (CO3)

(OR)

7 . Prove that by mathematical induction  $3n < n!$  whenever n is a positive integer greater than 6? (CO3)

8 Find the solution to the recurrence relation  $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$  with initial conditions  $a_0 = 2$ ,  $a_1 = 5$  and  $a_2 = 15$

(CO4)

(OR)

9 . a) Write short notes on Recurrence Relation.

b) Find the number of permutations of the letters of TOMANDJERRY so that same letters do not appear together.

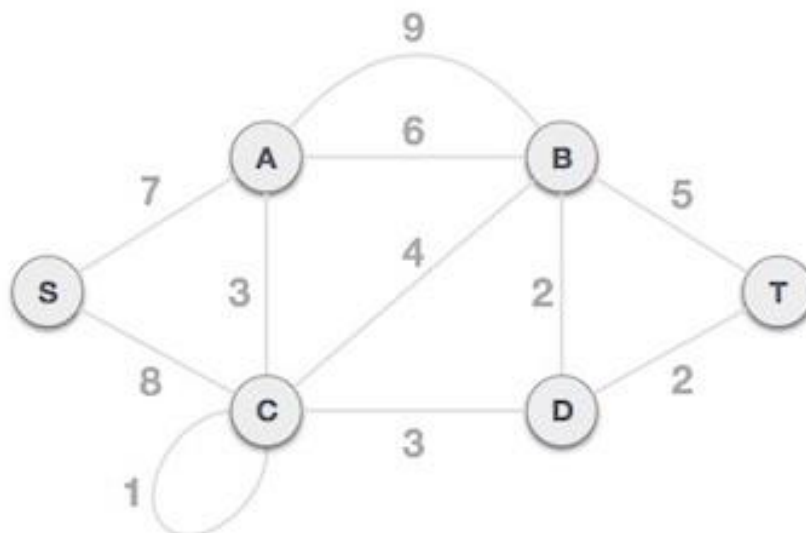
(CO4)

10 . Explain the steps involved in deriving a spanning tree from the given undirected graph using Breadth First Search algorithm, with an Example.

(CO5)

(OR)

11 Find the Minimum Cost of the Spanning Tree of the following Graph, by using Kruskal's algorithm



(CO5)

## Mid-1I Scheme of evaluation

### DEPARTMENT OF CSE

COURSE: **B.Tech**

YEAR: **II**

SEM: **I**

A-Y: **2021-22**

NAME OF SUBJECT: **DISCRETE MATHEMATICS**

MID: **I**

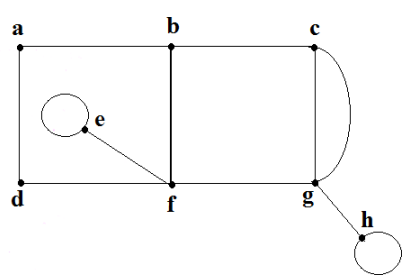
Max marks: **30**

### DESCRIPTIVE TEST

Time : 10.00 AM TO 11-30 AM.

Part A questions are all mandatory. Part B attempt any 3 question from any unit

### SCHEME OF EVALUATION

S.No	THEORY	MARKS	TOTAL
Part-A	a . Explain about the Strong Induction	2	2
	b . Discuss the following terms: (i)mean	1	2
	(ii)variance	1	
	c State and prove the Bayes Theorem?	2	2
	d a)Define Spanning tree . b) Find the degree of each region in the following Planar graph 	2	2
	e Define Bipartite graph. Discuss with an example.	2	2
Part-B	Q1. Prove by Mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n.	5	5
	Prove by Mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n.	5	5
	Q3 Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$	5	
	Q4 Explain the steps involved in deriving a spanning tree from the given undirected graph using Breadth First Search algorithm, with an	5	5

		Example.			
	Q5.	a) Write short notes on Recurrence Relation	2.5		
		b) ) Find the number of permutations of the letters of TOMANDJERRY so that same letters do not appear together.	2.5	5	
	Q6	<p>11 Find the Minimum Cost of the Spanning Tree of the following Graph, by using Kruskal's algorithm</p>	5	5	
		<b>DESCRIPTIVE TEST</b>			25 Marks
		ASSIGNMENT			5Marks
		TOTAL			30 Marks

## 11.Mapping of Course Objectives, Course Outcomes with PEOs and Pos

	Program Outcome(PO):												
PEOS		1	2	3	4	5	6	7	8	9	10	11	12
	I	x	x	x	x								
	II	x	x	x	x	x						x	
	III		x	x	x	x							

Course Outcomes	Relationship of Course outcomes to Program Outcomes (PO AVG)													
CO/PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO 1	PS O2
CO1	3	3	2	3	-	-	-	-	-	-	-	2	-	2
CO2	2	3	3	3	-	-	-	-	-	-	-	3	1	3
CO3	3	3	3	3	-	1	-	-	-	-	-	2	-	2
CO4	3	2	2	2	-	-	-	-	-	-	-	2	2	3
CO5	3	3	3	3	-	-	-	-	-	1	-	3	2	3

### Justification:

**CO1: Understand and construct precise mathematical proofs (Apply)**

**Correlated with PO1 high:** Because it contributes the knowledge on fundamentals of Mathematical logic which makes students get engineering knowledge and student can categorize different types of logics. So, overall the correlation of CO1 to PO1 is high.

**Correlated with PO2 high:** as this course outcome provides students identify different types of logic differences contribute a solution to research problems, Complex Problems. So, overall the correlation of CO1 is high.

**Correlated with PO3 moderately:** It contributes to identify the problems that arises but, cannot provide a complete solution to Complex problems. So, overall the correlation of CO1 is moderate.

**Correlated with PO4 high:** It contributes to identify the problems that provide a complete solution to Complex problems. So, overall the correlation of CO1 is high.

**Correlated with PO12 moderate:** It contributes to identify the problems that provide a complete solution to Complex problems. So, overall the correlation of CO1 is moderate.

**Correlated with PSO2 moderate:** It contributes to identify the problems that arises but, cannot provide a complete solution to Complex problems. So, overall the correlation of CO1 is moderate.

**CO2: Ability to use logic and set theory to formulate precise statements (Create)**

**Correlated with PO1 moderately:** Because it provides fundamentals of set theory to formulate precise statements So, correlation is average.

**Correlated with PO2 high:** contribution to provide solutions for logics to formulate precise statements So, correlation is high.

**Correlated with PO3 high:** contribution to develop solutions for logics to formulate precise statements So, correlation is high.

**Correlated with PO4 high:** contribution to conduct complex problems and solutions for logics to formulate precise statements So, correlation is high

**Correlated with PO12 high:** It contributes to provide scope of identifying solution to complex problems as a lifelong learning process. So, the correlation of CO2 is high.

**CO3: Analyze and solve counting problems on finite and discrete structures (Analyze)**

**Correlated with PO1 high:** contribution of this course outcome is high for providing counting problems of discrete structures i.e. in research area. The correlation is high.

**Correlated with PO2 high:** the CO contributes knowledge on different techniques on Permutations and combinations such that the student gets knowledge on using different counting approach. So, the correlation of CO is high.

**Correlated with PO3 high:** the CO contributes knowledge on designing solutions on Permutations and combinations such that the student gets knowledge on using different counting approach. So, the correlation of CO is high.

**Correlated with PO4 high:** Students get knowledge on investigations on complex problems of permutations and combinations using Formulas. The correlation is high.

**Correlated with PO6 low:** Students get knowledge on complex problems of permutations and combinations using Formulas but not completed So correlation is low.

**Correlated with PO12 moderate:** Students get knowledge on complex problems of permutations and combinations using Formulas but not completed So correlation is moderate.

**Correlated with PSO2 moderate:** Students get knowledge on different techniques but cannot

apply directly in research. The correlation is moderate.

**CO4: Elaborate** and manipulate sequences and advanced computing techniques (Create)

**Correlated with PO1 high:** Because it contributes the knowledge on fundamentals of mathematical logic which makes students get engineering knowledge and student can categorize Computational techniques. So, overall the correlation of CO4 to PO1 is high.

**Correlated with PO2 moderately:** as this course outcome provides students identify different Problems that occur when dealing with processes but cannot provide better solution for solving the issues So, overall the correlation of CO4 is moderate.

**Correlated with PO3 moderately:** as this course outcome provides students identify sequences but cannot provide better solution. So correlation is moderate.

**Correlated with PO4 moderate:** It contributes only knowledge on developing complex problems but, cannot provide a complete solution for all the counting techniques. So, overall the correlation of CO4 is average.

**Correlated with PO12 moderate:** It contributes only knowledge on developing complex problems on advanced computing techniques as lifelong learning. So, overall the correlation of CO4 is moderate.

**Correlated with PSO1 moderate** It contributes only knowledge on developing complex problems but, cannot provide a complete solution to Complex problems. So, overall the correlation of CO4 is moderate.

**Correlated with PSO2 high:** It contributes only knowledge on developing finite sequences. So, overall the correlation of CO4 is high.

**CO5: Apply** graph theory in solving computing problems (Apply)

**Correlated with PO1 high:** Because it contributes the knowledge on fundamentals of Graphs which makes students get engineering knowledge and student can categorize different nontrivial Problems. So, overall the correlation of CO5 to PO1 is high.

**Correlated with PO2 high:** as this course outcome provides students identify different Problems that occur when dealing with Nontrivial Problems provide better solution for solving the issues as analyzing problem. So, overall the correlation of CO5 is high.

**Correlated with PO3 high:** outcome contributes better for identification of different solutions for problems and develop solutions. So, that the students can apply to build some applications. So the correlation is high.

**Correlated with PO4 high:** outcome contributes better for identification of different solutions for problems. So, that the students can apply to build some applications. So the correlation is high.

**Correlated with PO12 moderate:** outcome contributes better for identification of different

solutions for problems as lifelong learning process but couldn't achieve some extent. So, Correlation is moderate.

**Correlated with PSO1 Moderate:** outcome contributes better for identification of graph theory. So, that the students can apply to build some applications. So the correlation is average.

**Correlated with PSO2 high:** outcome contributes better for identification of graph theory on computing problems. So, that the students can apply to build some applications. So the correlation is high.



**12. ATTAINMENT OF CO's, PO's AND PSO's (EXCEL SHEET):**

Relationship of Course outcomes to Program Outcomes (PO AVG)													
PO1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO1 0	PO1 1	PO1 2	PSO 1	PSO 2

ASSESSMENT OF POs THROUGH THE COURSE			
PO	CO	Value	AVG PO (mid)
PO1	CO1		
	CO3		
	CO4		
	CO5		
	CO6		
PO2	CO1		
	CO2		
	CO3		
	CO4		
	CO5		
PO3	CO1		
	CO2		
	CO3		
	CO4		
	CO5		
	CO6		
PO4	CO2		
	CO3		
	CO6		
PO5	CO5		
PO12	CO2		
PSO1	CO2		
	CO3		
PSO2	CO1		
	CO2		
	CO3		
	CO4		
	CO6		

### 13. University Question Papers or Question Bank.

Code No: 154AQ

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY  
HYDERABAD

B.Tech II Year II Semester Examinations, November/December - 2020

DISCRETE MATHEMATICS (Common to CSE, IT)

Time: 2 hours

Max.

Marks: 75

Answer any five questions All questions carry equal marks - - -

1.a) Show that  $\sim p \vee (\sim p \wedge q)$  and  $(\sim p \wedge \sim q)$  are logically equivalent.

b) Show that  $\sim p \wedge (\sim q \vee r) \Rightarrow \sim p$ .

[8+7]

2. Prove that  $\forall x P x \vee Q x \Rightarrow x P x \vee \exists x (x)$ .

[15]

3. Show that congruence modulo  $m$  is an equivalence relation on integers.

[15]

4.a) A relation  $R$  on  $A$  is symmetric if and only if  $R = R^{-1}$ .

b) A relation  $R$  on  $A$  is reflexive if and only if  $R^{-1}$  is reflexive.

[7+8]

5. Prove by Mathematical induction that  $6n^2 + 7n + 1$  is divisible by 43 for each positive integer  $n$ . [15]

6. Prove that, if  $F_n$  is the  $n$ th Fibonacci number, then  $F_n = \frac{1}{\sqrt{5}} \left( \frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left( \frac{1-\sqrt{5}}{2} \right)^n$  for all integers  $n \geq 0$ . [15]

7. Solve the recurrence relation  $a_n - a_{n-1} - 12a_{n-2} = 0$ ,  $a_0 = 0$ ,  $a_1 = 1$ . [15]

8.a) State and prove fundamental theorem of graph theory.

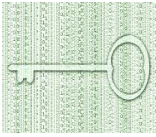
b) Prove that a complete graph  $K_n$  is planar if and only if  $n \leq 4$ . [7+8]

--ooOoo--

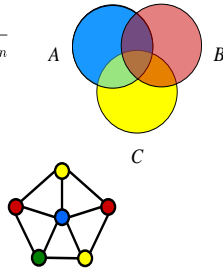
## 14.PPTs AND PRESENTATION

### Introduction to Discrete Mathematics

$$\frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$



$$a = qb + r \iff \gcd(a, b) = \gcd(b, r)$$



### Course Material

• Textbook: Discrete Mathematics and its Applications, 7<sup>th</sup> ed  
Author: Kenneth H. Rosen  
Publisher: McGraw Hill

• Reference Texts (links available at the course-page):  
▪ Course notes from "mathematics for computer science"  
▪ Discrete Mathematics, Lecture Notes, by L. Lovász and K. Vesztegombi

### Example 1

How to play Rubik Cube?

Google: Rubik cube in 26 steps



<http://www.cse.cuhk.edu.hk/~chi/csc2110-2008/notes/Rubik-Cube.ppt>

### Basic Information

- **Course homepage:**  
<http://www.cse.iitd.ac.in/~naveen/teaching/courses/COL202/>
- **Instructor:** Naveen Garg
- **Teaching Assistants:** Jatin Batra, Nikhil Kumar
- **Lectures:** M(0800-0920) and Th(0800-0920)
- **Tutorials:** M, W, Th (1300-1400)
- **Slides:**
  - Will be posted on the course page
  - adapted (with permission from Lac chi Lau) from course on Discrete Mathematics at CUHK.

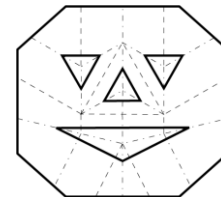
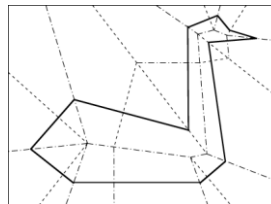
### Course Requirements

- Minors: 30%
- Lecture Quizzes: 25%
- Tutorial Quizzes: 10%
- Major 35%

### Example 2

The mathematics of paper folding

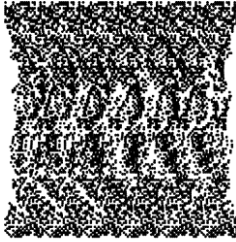
<http://www.ushistory.org/betsy/flagstar.html>



<http://erikdemaine.org/foldcut/>

### Example 3

3D-images



• <http://128.100.68.6/~drorbn/papers/PDI/>

### Why Mathematics?

Design efficient computer systems.

- How did Google manage to build a fast search engine?
- What is the foundation of internet security?

algorithms, data structures, database,  
parallel computing, distributed systems,  
cryptography, computer networks...

Logic, number theory, counting, graph theory...

### Topic 1: Logic and Proofs

How do computers think?

**Logic:** propositional logic, first order logic

**Proof:** induction, contradiction

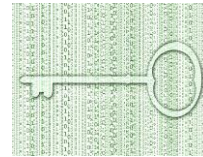
$$\frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$



Artificial intelligence, database, circuit, algorithms

### Topic 2: Number Theory

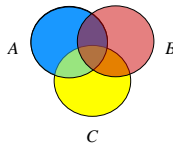
- Number sequence
- (Extended) Euclidean algorithm
- Prime number, modular arithmetic, Chinese remainder theorem
- Cryptography, RSA protocol



Cryptography, coding theory, data structures

### Topic 3: Counting

- Sets and Functions
- Combinations, Permutations, Binomial theorem
- Counting by mapping, pigeonhole principle
- Recursions



Probability, algorithms, data structures

### Topic 3: Counting

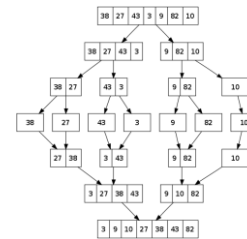
How many steps are needed to sort n numbers?

#### Algorithm 1 (Bubble Sort):

Every iteration moves the i-th smallest number to the i-th position

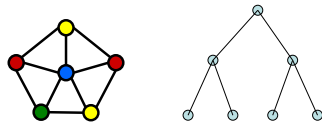
#### Algorithm 2 (Merge Sort):

Which algorithm runs faster?



## Topic 4: Graph Theory

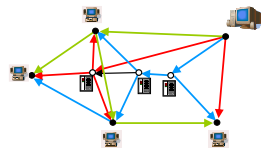
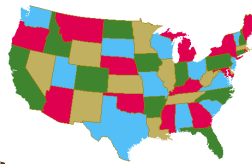
- Graphs, Relations
- Degree sequence, Eulerian graphs, isomorphism
- Trees
- Matching
- Coloring



Computer networks, circuit design, data structures

## Topic 4: Graph Theory

How to color a map?

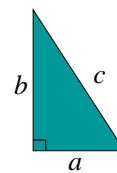


How to send data efficiently?

## Objectives of This Course

- To learn basic mathematical concepts, e.g. sets, functions, graphs
- To be familiar with formal mathematical reasoning, e.g. logic, proofs
- To improve problem solving skills
- To see the connections between discrete mathematics and computer science

## Pythagorean theorem

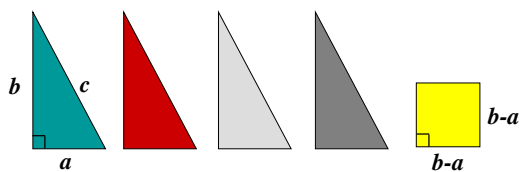


$$a^2 + b^2 = c^2$$

Familiar?

Obvious?

## Good Proof



We will show that these five pieces can be rearranged into:

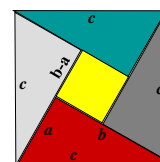
- a  $c \times c$  square, and then
- an  $a \times a$  & a  $b \times b$  square

And then we can conclude that  $c^2 = a^2 + b^2$

## Good Proof

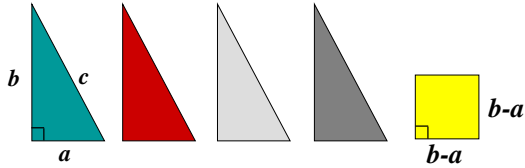
The five pieces can be rearranged into:

- a  $c \times c$  square

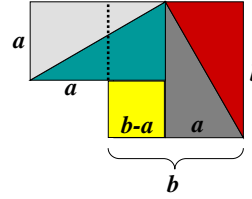


### Good Proof

How to rearrange them into an  $a \times a$  square and a  $b \times b$  square?



### Good Proof

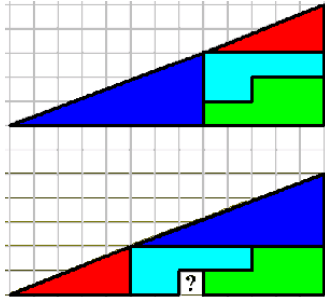


74 proofs in <http://www.cut-the-knot.org/pythagoras/index.shtml>

### Bad Proof

A similar rearrangement technique shows that  $65=64$ ...

What's wrong with the proof?



### Mathematical Proof

To prove mathematical theorems, we need a more rigorous system.

The standard procedure for proving mathematical theorems is invented by Euclid in 300BC. First he started with five **axioms** (the truth of these statements are taken for granted). Then he uses **logic** to deduce the truth of other statements.

1. It is possible to draw a **straight line** from any point to any other point.
2. It is possible to produce a **finite** straight line continuously in a straight line.
3. It is possible to describe a **circle** with any center and any radius.
4. It is true that all **right angles** are equal to one another.
5. ("Parallel postulate") It is true that, if a straight line falling on two straight lines make the **interior angles** on the same side less than two right angles, the two straight lines, if produced indefinitely, **intersect** on that side on which are the **angles** less than two right angles.

Euclid's proof of Pythagorean's theorem [http://en.wikipedia.org/wiki/Pythagorean\\_theorem](http://en.wikipedia.org/wiki/Pythagorean_theorem)

### Statement (Proposition)

A **Statement** is a sentence that is either **True** or **False**

Examples:  $2 + 2 = 4$  **True**  
 $3 \times 3 = 8$  **False**  
 787009911 is a prime

Non-examples:  $x + y > 0$   
 $x^2 + y^2 = z^2$

They are true for some values of  $x$  and  $y$  but are false for some other values of  $x$  and  $y$ .

### Logic Operators

$\neg$  **::= NOT**

$\sim p$  is true if  $p$  is false

$\wedge$  **::= AND**

$\vee$  **::= OR**

$P$	$Q$	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

$P$	$Q$	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

## Compound Statement

p = "it is hot"

q = "it is sunny"

It is hot and sunny

$$p \wedge q$$

It is not hot but sunny

$$\neg p \wedge q$$

It is neither hot nor sunny

$$\neg p \wedge \neg q$$

## Exclusive-Or

coffee "or" tea  $\leftarrow \oplus$  exclusive-or

How to construct a compound statement for exclusive-or?

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Idea 1: Look at the true rows

Want the formula to be true exactly when the input belongs to a "true" row.

$$(p \wedge \neg q) \vee (\neg p \wedge q)$$

The input is the second row exactly if this sub-formula is satisfied

And the formula is true exactly when the input is the second row **or** the third row.

## Exclusive-Or

coffee "or" tea  $\leftarrow \oplus$  exclusive-or

How to construct a compound statement for exclusive-or?

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Idea 2: Look at the false rows

Want the formula to be true exactly when the input does **not** belong to a "false" row.

$$\neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q)$$

The input is the first row exactly if this sub-formula is satisfied

And the formula is true exactly when the input is **not** in the 1<sup>st</sup> row **and** the 4<sup>th</sup> row.

## Logical Equivalence

Idea 3: Guess and check

$$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$$

p	q	$p \oplus q$	$p \vee q$	$\neg(p \wedge q)$
T	T	F	T	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

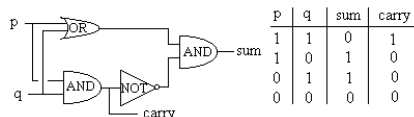
Logical equivalence: Two statements have the same truth table

As you see, there are many different ways to write the same logical formula.

One can always use a truth table to check whether two statements are equivalent.

## Writing Logical Formula for a Truth Table

Digital logic:



Given a digital circuit, we can construct the truth table.

Now, suppose we are given only the truth table (i.e. the specification), how can we construct a circuit (i.e. formula) that has the same function?

## Writing Logical Formula for a Truth Table

Use idea 1 or idea 2.

Idea 1: Look at the true rows and take the "or".

p	q	r	output
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

$$(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r)$$

$$\vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$$

The formula is true exactly when the input is one of the true rows.

## Writing Logical Formula for a Truth Table

	p	q	r	output	
$p \wedge q \wedge r$	T	T	T	F	$\neg(p \wedge q \wedge r)$
$p \wedge q \wedge \neg r$	T	T	F	T	
$p \wedge \neg q \wedge r$	T	F	T	T	
$p \wedge \neg q \wedge \neg r$	T	F	F	F	$\wedge \neg(p \wedge \neg q \wedge \neg r)$
$\neg p \wedge q \wedge r$	F	T	T	T	
$\neg p \wedge q \wedge \neg r$	F	T	F	T	
$\neg p \wedge \neg q \wedge r$	F	F	T	T	
$\neg p \wedge \neg q \wedge \neg r$	F	F	F	F	$\wedge \neg(\neg p \wedge \neg q \wedge \neg r)$

Idea 2: Look at the false rows, negate and take the "and".

The formula is true exactly when the input is not one of the false row.

## DeMorgan's Laws

Logical equivalence: Two statements have the same truth table

De Morgan's Law

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Statement: Tom is in the football team and the basketball team.

Negation: Tom is not in the football team or not in the basketball team.

De Morgan's Law

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Statement: The number 783477841 is divisible by 7 or 11.

Negation: The number 783477841 is not divisible by 7 and not divisible by 11.

## DeMorgan's Laws

Logical equivalence: Two statements have the same truth table

De Morgan's Law

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

De Morgan's Law

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

## Simplifying Statement

$$\begin{aligned} & \neg(\neg p \wedge q) \wedge (p \vee q) \\ \equiv & (\neg\neg p \vee \neg q) \wedge (p \vee q) \\ \equiv & (p \vee \neg q) \wedge (p \vee q) \\ \equiv & p \vee (\neg q \wedge q) \\ \equiv & p \vee \text{False} \\ \equiv & p \end{aligned}$$

DeMorgan

Distributive

See textbook for more identities.

## Tautology, Contradiction

A tautology is a statement that is always true.

$$\begin{aligned} & p \vee \neg p \\ & (p \wedge q) \vee (\neg q \wedge p) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge q) \end{aligned}$$

A contradiction is a statement that is always false. (negation of a tautology)

$$\begin{aligned} & p \wedge \neg p \\ & (p \vee q) \wedge (\neg q \vee p) \wedge (\neg p \vee \neg q) \wedge (\neg p \vee q) \\ & ((p \wedge r) \vee (q \wedge r)) \wedge (\neg(p \vee q) \vee r) \end{aligned}$$

In general it is "difficult" to tell whether a statement is a contradiction. It is one of the most important problems in CS - the satisfiability problem.

## Quick Summary

Key points to know.

1. Write a logical formula from a truth table.
2. Check logical equivalence of two logical formulas.
3. DeMorgan's rule and other simple logical rules (e.g. distributive).
4. Use simple logical rules to simplify a logical formula.



### Course Project

Pick an interesting mathematical topic, write a report of about 10 pages.

3 students in a group

Can use any references, but cite them.

Choose 1-3 groups to present, up to 5% bonus

### A Project

Tell an interesting story related to mathematics.

More about good topic and nice presentation, than mathematical difficulty.

- Interesting or curious problems, interesting history
- Surprising or elegant solutions
- Nice presentation, easy to understand

### Project Ideas

- Magic tricks
- More games, more paper folding, etc
- Logic paradoxes
- Prime numbers
- Game theory

<http://www.cse.cuhk.edu.hk/~chi/csc2110/project.html>

## 15. Websites or URLs e- Resources

1. [https://www.tutorialspoint.com/discrete\\_mathematics/index.htm#:~:text=Discrete%20Mathematics%20is%20a%20branch,reasoning%20and%20problem%2Dsolving%20capabilities](https://www.tutorialspoint.com/discrete_mathematics/index.htm#:~:text=Discrete%20Mathematics%20is%20a%20branch,reasoning%20and%20problem%2Dsolving%20capabilities)
2. <https://nptel.ac.in/courses/111/107/111107058/>
3. <https://nptel.ac.in/courses/106/106/106106094/>